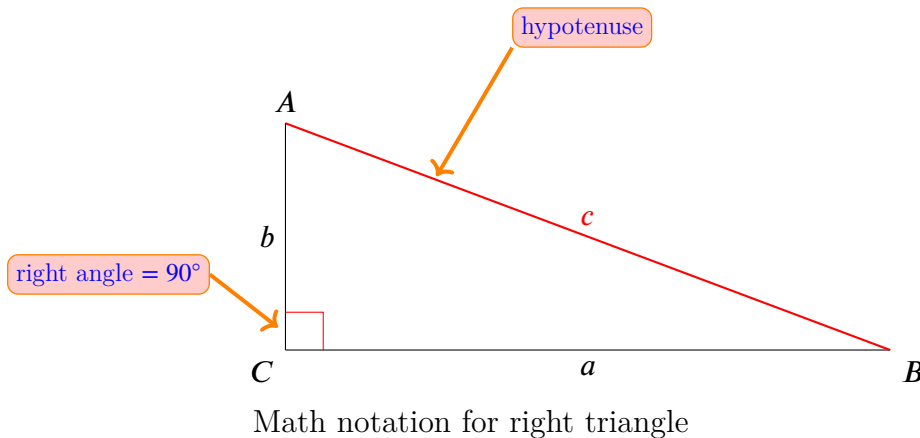


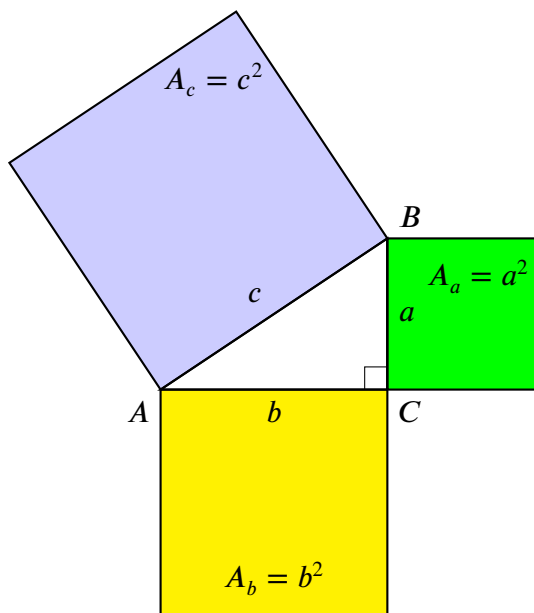
4.4. Pythagorean Theorem

A **right triangle** is a triangle containing a right angle (90°). A triangle cannot have more than one right angle, since the sum of the two right angles plus the third angle would exceed the 180° total possessed by a triangle. The side opposite the right angle is called the hypotenuse (side c in the figure below). The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).



The Pythagorean Theorem states that the square of a hypotenuse is equal to the sum of the squares of the other two sides. It is one of the fundamental relations in Euclidean geometry.

$$c^2 = a^2 + b^2$$

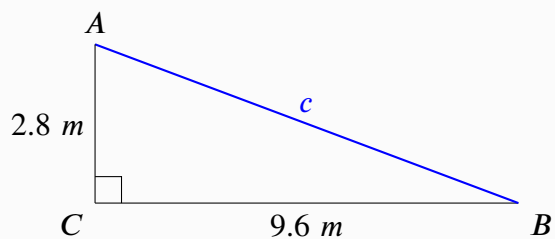


Pythagorean triangle with the squares of its sides and labels

Pythagorean triples are integer values of a , b , c satisfying this equation.

Finding the Sides of a Right Triangle

Example 1: Find the hypotenuse. 🏠



$$c^2 = a^2 + b^2 \quad \text{|substitute for } a \text{ and } b$$

$$c^2 = (9.6 \text{ m})^2 + (2.8 \text{ m})^2$$

$$c^2 = 92.16 \text{ m}^2 + 7.84 \text{ m}^2$$

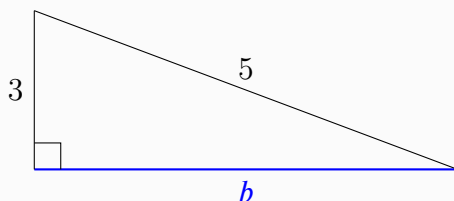
$$c^2 = 100 \text{ m}^2 \quad \text{|take the square root of each side}$$

$$\sqrt{c^2} = \sqrt{100 \text{ m}^2}$$

$$\sqrt{c^2} = \sqrt{100} \sqrt{\text{m}^2}$$

$$c = 10 \text{ m}$$

Example 2: Find the missing side b .



$$c^2 = a^2 + b^2 \quad \text{|subtract } a^2 \text{ from each side}$$

$$c^2 - a^2 = \cancel{a^2} + b^2 - \cancel{a^2}$$

$$c^2 - a^2 = b^2 \quad \text{|switch sides}$$

$$b^2 = c^2 - a^2 \quad \text{|substitute for } a \text{ and } c$$

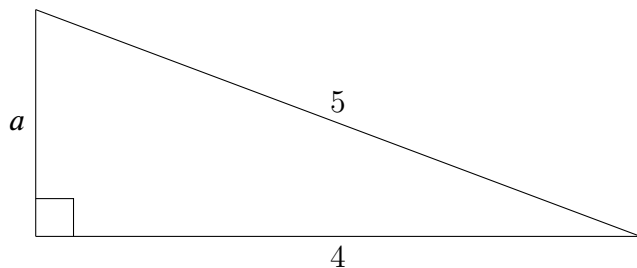
$$b^2 = 5^2 - 3^2$$

$$b^2 = 25 - 9 = 16 \quad \text{|take the square root of each side}$$

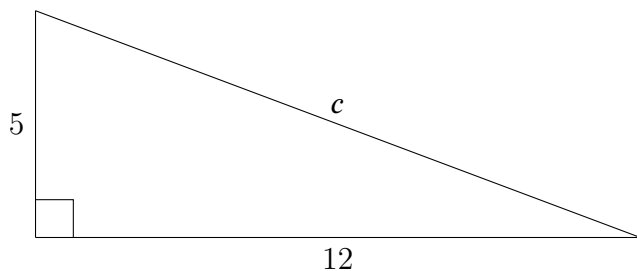
$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$

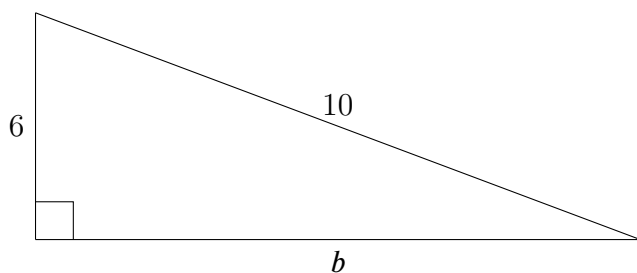
Practice 1: Find all the missing sides in each right angle triangle.



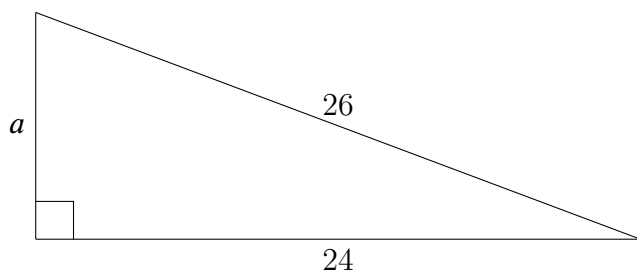
$$\begin{aligned}c^2 &= a^2 + b^2 \\a^2 &= c^2 - b^2 \\a^2 &= 5^2 - 4^2 = 25 - 16 = 9 \\\sqrt{a^2} &= \sqrt{9} \\a &= 3\end{aligned}$$



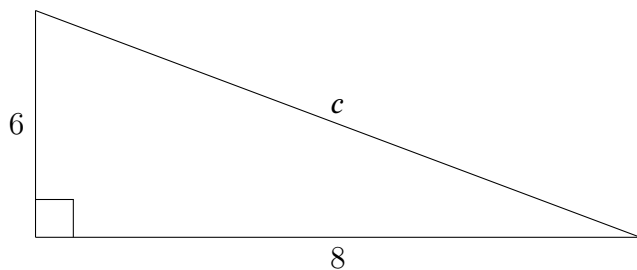
$$\begin{aligned}c^2 &= a^2 + b^2 \\c^2 &= 5^2 + 12^2 = 25 + 144 = 169 \\\sqrt{c^2} &= \sqrt{169} \\c &= 13\end{aligned}$$



$$\begin{aligned}c^2 &= a^2 + b^2 \\b^2 &= c^2 - a^2 \\b^2 &= 10^2 - 6^2 = 100 - 36 = 64 \\\sqrt{b^2} &= \sqrt{64} \\b &= 8\end{aligned}$$

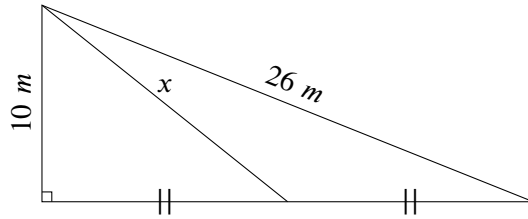


$$\begin{aligned}c^2 &= a^2 + b^2 \\a^2 &= c^2 - b^2 \\a^2 &= 26^2 - 24^2 = 676 - 576 = 100 \\\sqrt{a^2} &= \sqrt{100} \\a &= 10\end{aligned}$$



$$\begin{aligned}c^2 &= a^2 + b^2 \\c^2 &= 6^2 + 8^2 = 36 + 64 = 100 \\\sqrt{c^2} &= \sqrt{100} \\c &= 10\end{aligned}$$

Challenge 1: Find x . 🧮 😊



Use the Pythagorean theorem to find the base b in the larger triangle:

Given: hypotenuse : $c = 26\text{ m}$ $a = 10\text{ m}$ Find: $b = ?$

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{|subtract } a^2 \text{ from both sides} \\ b^2 &= (26\text{ m})^2 - (10\text{ m})^2 && \text{|substitute values } c = 26\text{ m} \text{ and } a = 10\text{ m} \\ b^2 &= 676\text{ m}^2 - 100\text{ m}^2 \\ b^2 &= 576\text{ m}^2 \\ \sqrt{b^2} &= \sqrt{576\text{ m}^2} && \text{|take the square root of each side} \\ b &= 24\text{ m} \end{aligned}$$

In the smaller triangle, the base (let's call it y) is half the base of the larger triangle.

Given: $y = \frac{1}{2}b = \frac{1}{2}(24\text{ m}) = 12\text{ m}$ $a = 10\text{ m}$ Find: hypotenuse : $x = ?$

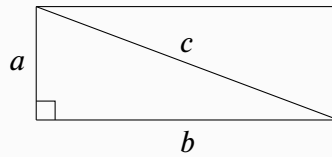
Use the Pythagorean theorem:

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{|substitute: } c = x, a = a \text{ and } b = y \\ x^2 &= a^2 + y^2 && \text{|substitute values: } a = 10\text{ m} \text{ and } y = 12\text{ m} \\ x^2 &= (10\text{ m})^2 + (12\text{ m})^2 \\ x^2 &= 100\text{ m}^2 + 144\text{ m}^2 \\ x^2 &= 244\text{ m}^2 && \text{|take the square root of each side} \\ \sqrt{x^2} &= \sqrt{244\text{ m}^2} \\ x &= 15.6\text{ m} \end{aligned}$$

Area of Right Triangle

The area of a triangle is equal to one half the base multiplied by the corresponding height: $A = \frac{bh}{2}$

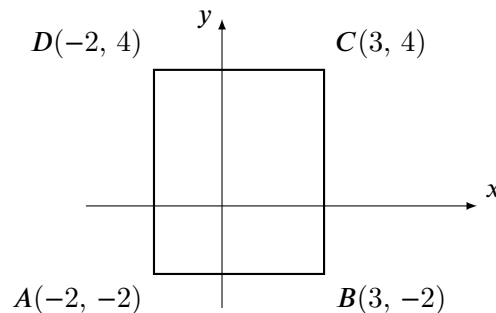
Example 3: Find the length of the diagonal of a rectangle that has width $a = 3$ and length $b = 4$.



The diagonal of a rectangle is the hypotenuse of a right triangle. Use the Pythagorean Theorem.

$$\begin{aligned}c^2 &= a^2 + b^2 \\c^2 &= 3^2 + 4^2 = 9 + 16 = 25 \\ \sqrt{c^2} &= \sqrt{25} \\ c &= 5 \quad \text{The length of the diagonal is } c = 5.\end{aligned}$$

Challenge 2: What is the length BD shown in the figure? 🗺️ 😊



Points A , B , and D form the right triangle with hypotenuse BD .

The length AB equals to change in x , from -2 in $A(-2, -2)$ to 3 in $B(3, -2)$, since points A and B have a fixed y coordinate.

$$AB = \Delta x = 3 - (-2) = 3 + 2 = 5$$

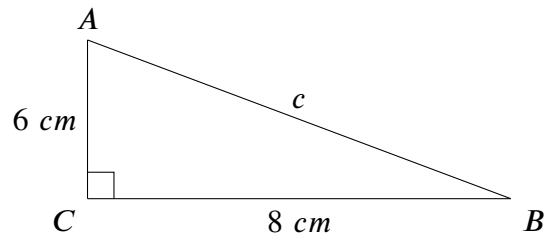
The length AD equals to change in y , from -2 in $A(-2, -2)$ to 4 in $D(-2, 4)$, since points A and D have a fixed x coordinate.

$$AD = \Delta y = 4 - (-2) = 4 + 2 = 6$$

Use the Pythagorean theorem to find length BD , which is a hypotenuse of the right triangle BAD .

$$\begin{aligned}BD^2 &= 5^2 + 6^2 = 25 + 36 \\ BD^2 &= 61 \\ \sqrt{BD^2} &= \sqrt{61} \\ BD &\approx 7.8\end{aligned}$$

Challenge 3: Find the area of a square whose perimeter is the same as the perimeter of the triangle shown below. 😊



To find the perimeter of the triangle we need to find the hypotenuse, side c , first.

$$P = a + b + c$$

The hypotenuse c of the triangle is

$$\begin{aligned}c^2 &= (6 \text{ cm})^2 + (8 \text{ cm})^2 \\&= 36 \text{ cm}^2 + 64 \text{ cm}^2 \\&= 100 \text{ cm}^2\end{aligned}$$

Take square root of both sides

$$\begin{aligned}\sqrt{c^2} &= \sqrt{100 \text{ cm}^2} \\c &= 10 \text{ cm}\end{aligned}$$

The perimeter of triangle P is the sum of the lengths of all three sides of the triangle:

$$P = 6 \text{ cm} + 8 \text{ cm} + 10 \text{ cm} = 24 \text{ cm}$$

Perimeters of the triangle and the square are same. If the side of the square is a , then the perimeter of the square is

$$P = 4a$$

Divide by 4 and switch sides

$$a = \frac{P}{4}$$

Substitute value for P

$$a = \frac{24 \text{ cm}}{4} = 6 \text{ cm}$$

Finally, the area of the square is

$$A_S = a^2 = (6 \text{ cm})^2 = 36 \text{ cm}^2$$