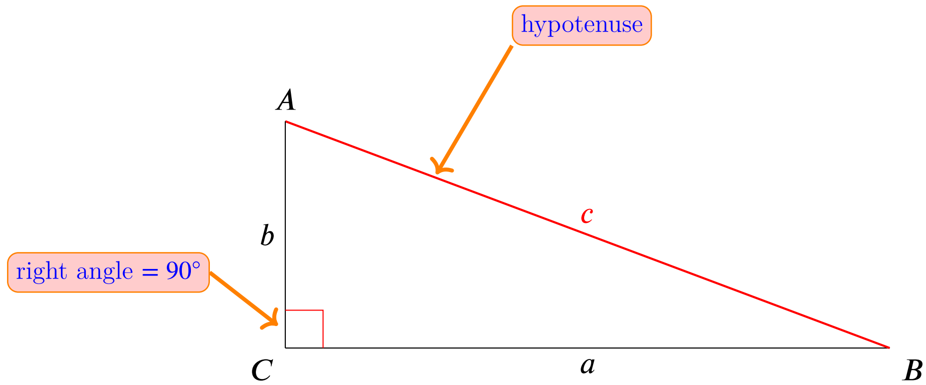
4.4. Pythagorean Theorem

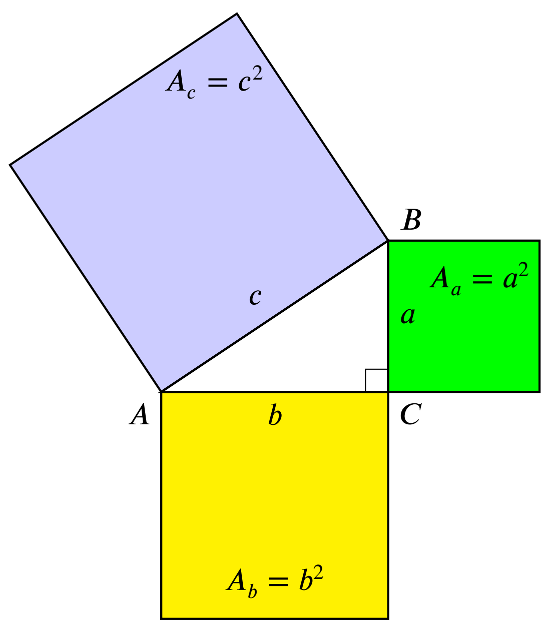
A **right-angle triangle** is a triangle containing a right angle (90°). A triangle cannot have more than one right angle, since the sum of the two right angles plus the third angle would exceed the 180° total possessed by a triangle. The side opposite the right angle is called the hypotenuse (side c in the figure below). The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).



Math notation for right-angle triangle

The Pythagorean Theorem states that the square of a hypotenuse is equal to the sum of the squares of the other two sides. It is one of the fundamental relations in Euclidean geometry.

$$c^2 = a^2 + b^2$$

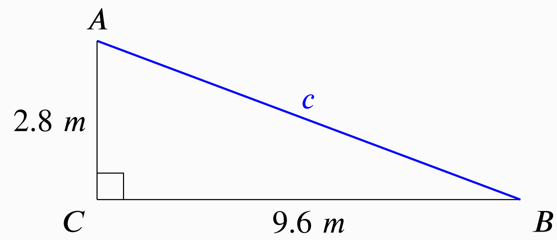


Pythagorean triangle with the squares of its sides and labels

Pythagorean triples are integer values of a, b, c satisfying this equation.

Finding the Sides of a Right Angled Triangle

Example 1: Find the hypotenuse.



$c^2 = a^2 + b^2$ |substitute for a and b

$c^2 = (9.6\ m)^2 + (2.8\ m)^2$

$c^2 = 92.16\ m^2 + 7.84\ m^2$

$c^2 = 100\ m^2$ |take the square root of each side

$\sqrt{c^2} = \sqrt{100\ m^2}$

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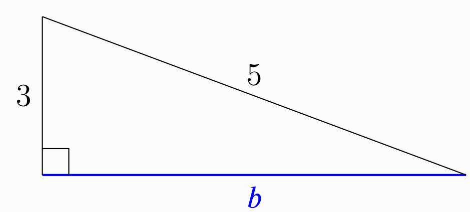
$c = 10\ m$

Example 2: Find the missing side b.



$c^2 = a^2 + b^2$ |subtract a^2 from each side

$c^2 - a^2 = \cancel{a^2} + b^2 \cancel{- a^2}$

$c^2 - a^2 = b^2$ |switch sides

$b^2 = c^2 - a^2$ |substitute for a and c

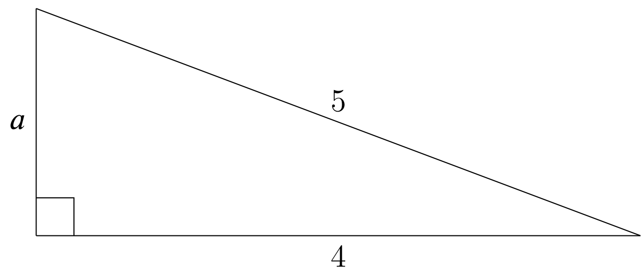
$b^2 = 5^2 - 3^2$

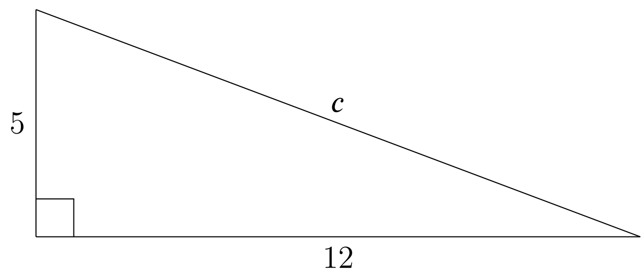
$b^2 = 25 - 9 = 16$ |take the square root of each side

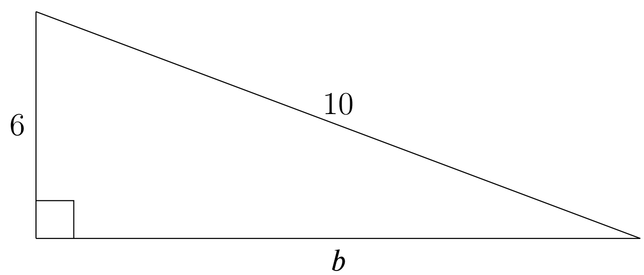
$\sqrt{b^2} = \sqrt{16}$

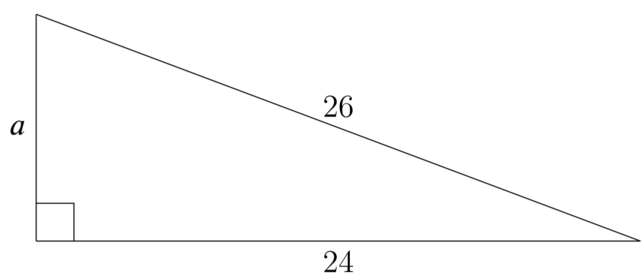
$b = 4$

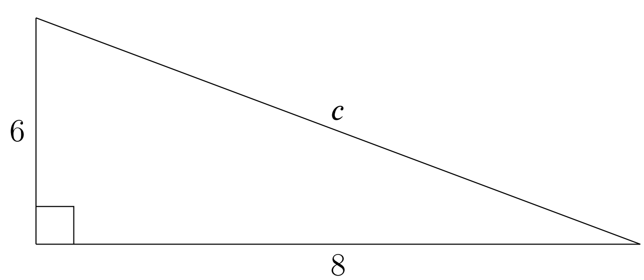
Practice 1: Find all the missing sides in each right angle triangle.



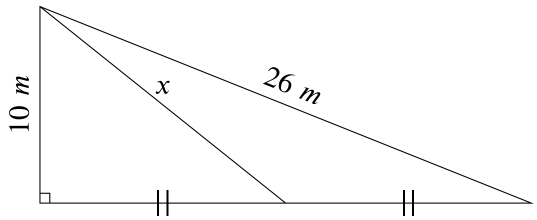








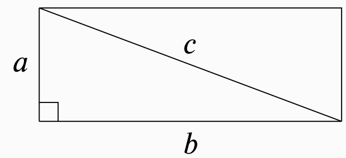
Challenge 1: Find x.



Area of Right Angled Triangle

The area of a triangle is equal to one half the base multiplied by the corresponding height: $A = \dfrac{bh}{2}$

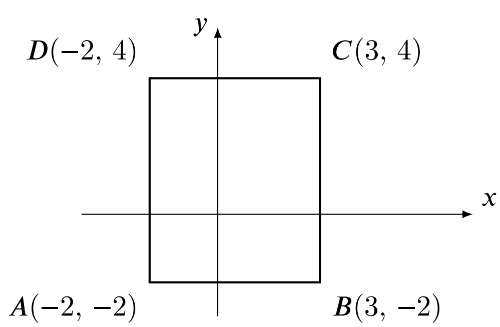
Example 3: Find the length of the diagonal of a rectangle that has width $a = 3$ and length $b = 4$.



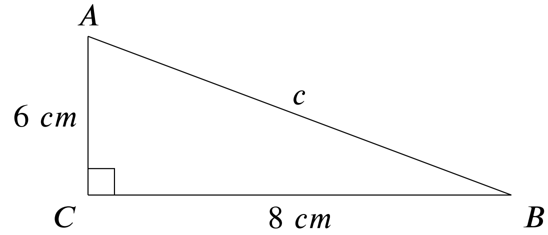
The diagonal of a rectangle is the hypotenuse of a right-angle triangle. Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5 \qquad \text{The length of the diagonal is } c = 5.$$

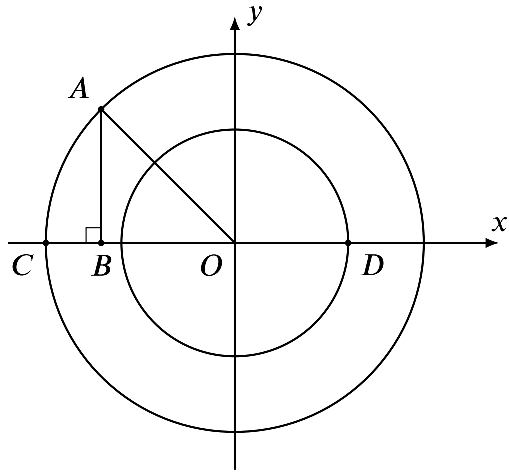
Challenge 2: What is the length BD shown in the figure?



Challenge 3: Find the area of a square whose perimeter is the same as the perimeter of the triangle shown below.



Challenge 4: Two concentric circles with the center at the origin are shown below. $A(4,\ 3)$ is on the larger circle and CD is 9, what is the radius of the smaller circle? 😐



Challenge 5: A rectangle with the area of $3\sqrt{2}$ is inscribed into a square with the side length of $a+b$, as shown below. Find the length of the rectangles' diagonal d, if $a : b = 2\sqrt{2} : 3$. ☹